LIQUID-FILM HEAT-TRANSFER COEFFICIENTS IN HEATING LINSEED AND PETROLEUM OIL

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Liquid-Film Heat Transfer Coefficients

For Condensing Dowtherm A Vapors to Petroleum and Linseed Oils in a Vertical-Tube Heater[1]

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A heavy petroleum oil and linseed oil were heated to temperatures up to 600° F. in a tubular heater, using Dowtherm vapor as the heating medium. The film coefficients for heating the petroleum oil agreed reasonably well with the Dittus and Boelter equation. This work is of importance because it confirms the Dittus and Boelter equation up to higher temperatures than have ever been used before (600° F.) and up to higher Reynolds numbers (50,000) than have ever been reached in the heating of oils.

Linseed oil, both raw and bodied, followed the Dittus and Boelter equation with a precision that would ordinarily be considered satisfactory. By trials with different exponents on both the Reynolds and the Prandtl groups, it was found that the equation

$$\frac{hD}{k} = 0.00725 \left(\frac{Du\rho}{\mu}\right)^{0.87} \left(\frac{C\mu}{k}\right)^{0.5}$$

represented the results on linseed oil with a high degree of precision. This work covers a range of the Reynolds group up to 90,000 and of the Prandtl group up to 370.

IN THE turbulent range and for vertical tubes, data are available for the film coefficient of heat transfer from metals to water and oil at moderate temperatures (*14, 15*). Logan, Fragen, and Badger (*10*) measured liquid-film heat transfer coefficients in a semicommercial vertical-tube forced-circulation evaporator. They boiled sugar solutions of different concentrations, and the data obtained could be correlated by using the Dittus and Boelter type of equation. They obtained the same exponents for the Reynolds and Prandtl groups as did Dittus and Boelter (*13*). The constant for the equation was 0.0205 as compared to 0.0225 obtained by Dittus and Boelter. The range of their Reynolds number was from 7800 to 250,000 and the Prandtl number from 2 to 50.

This investigation involves the study of liquid-film coefficients of heat transfer between condensing vapors of Dowtherm A and petroleum and linseed oils in a vertical-tube heater. Dowtherm A is the eutectic mixture of 73.5 per cent diphenyl oxide and 26.5 per cent diphenyl, and was described by Heindel (*8*). Holser (*9*) presented heat transfer data involving the use of this material. The work on the evaporation of caustic soda by Badger, Monrad, and Diamond (*1*) and the heating of asphalt by Montillon, Rohrbach, and Badger (*18*) was carried out with condensing diphenyl vapors. The advantage of Dowtherm A over either diphenyl oxide or diphenyl is its melting point of 53.6° F. A stable petroleum oil of high grade was used. It was obtained from the Kendall Refining Company and had a viscosity of 140 seconds at 210° F. The thermal properties of Dowtherm A were obtained from calculations and data of Ullock, Gaffert, Konz, and Brown (*25*).

Equipment Flow Sheet

The equipment flow sheet is shown in Figure 1. Its details were dictated primarily by the problem of bodying linseed oil: The oil was pumped from the flash pot to the heater, heated to about 580° F., and then went to a cooler where the temperature immediately dropped 10° or 20° F. From the cooler the oil was returned to the flash pot where an inert atmosphere and a reduced pressure were maintained and where any vapors formed were released; it was then returned to the pump. The heating and cooling were continued until the oil attained the desired characteristics of a bodied linseed oil.

No iron could be used where it would come in contact with the linseed oil because it would produce a dark color in the heated oil. Nickel, brass, and aluminum were used for pipes and fittings. The boiler that supplied the Dowtherm A vapors was a semicommercial type, gas-fired unit (*17*).

The vertical two-tube heater was approximately 12 feet long. The tubes were $^7/_8$ inch o. d., 16 Birmingham wire gage nickel tubes with a heating length of 11 feet 8.5 inches between the tube sheets, and were rolled into nickel-clad tube sheets. The shell of the heater was a 3-inch extra heavy steel pipe. The apparatus was to be subjected to temperatures around 700° F. This would produce stresses between the tubes and the tube sheet due to a difference in the expansion coefficients for the steel pipe and nickel tubes. The tubes were prevented from slipping when under stress by cutting grooves $^1/_{16}$ inch wide and deep in each tube sheet, and in the rolling operation the nickel tubes were expanded into these grooves.

Approximately 18 inches from each end of the heater a 4-inch nipple and a 10-inch flange were welded on opposite sides of the shell to form a cross and allow access to the nickel tubes in the heater (Figure 2). A 2-inch vapor inlet at the top and a 1.25-inch condensate outlet at the bottom were welded to the heater.

Thermocouple Installation

Copper-constantan and copel-X-chromel thermocouples were used for temperature measurements, and the copper-constantan couples were used at the lower temperatures. Liquid tempera-

[1] Complete tables of data obtained in these experiments will be published in the *Transactions of the American Institute of Chemical Engineers.*

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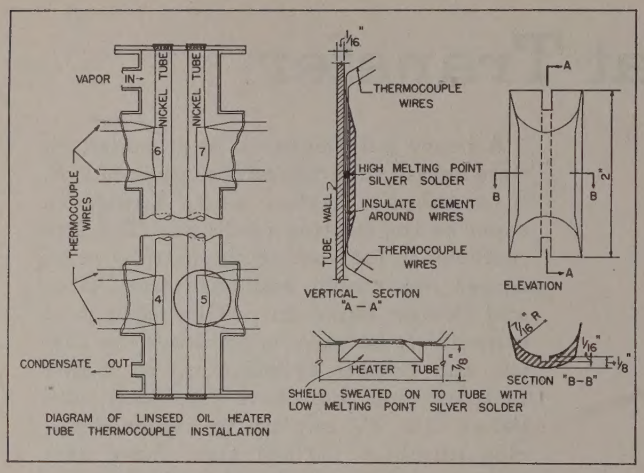

FIGURE 1. EQUIPMENT FLOW SHEET FOR HEAT TRANSFER TO BODYING LINSEED OIL

tures were measured with bare thermocouples specially installed in tees on the pipe line; the lead wires came through packing glands. The vapor temperatures were measured with couples that had their hot junctions silver-soldered into small blocks of brass. The brass block prevented small fluctuations in temperatures. The couples were installed to meet the conditions outlined by Colburn and Hougen (*4*). Care was taken to have the junction of the couples situated at the center of the stream. Mixing boxes were installed before each thermocouple so that the average temperature of the liquid stream was measured.

The tube temperatures were measured with thermocouples mounted on the tubes at the top and at the bottom of the heater as illustrated in Figure 2. It was not possible to use the method developed by Hebbard and Badger (*7*) but the combined suggestions of Hebbard and McCabe (*6, 16*) were used in the method illustrated in Figure 2. It consisted of a shield made from nickel tubing with an inside diameter equal to the outside diameter of the nickel tubes in the heater. Two-inch lengths of this tubing were cut in half along the axis of the tube. When these pieces were placed on the heater tubes, they fitted snugly. The edges of each shield were beveled off so as to minimize the effect of the shields on the layer of condensate flowing down over the surface of the heater tubes. A groove $^1/_8$ inch wide and $^1/_{16}$ inch deep was cut on the inside of the shield. The junctions of two couples were then soldered at the center of the shield in the groove with a silver solder of high melting point. The excess solder was filed off flush with the inside surface of the shield. An insulating cement was then applied around the thermocouple wires which lay stretched out in the groove of the shield and was allowed to dry. Later the excess cement was filed off flush with the inside surface of the shield until it fitted snugly on the heater tube. The shields were then sweated onto the outside wall of the heater tubes with a silver solder of low melting point.

The most serious objection to this method is the change in the surface of the tubes where the shields are attached. They change the characteristics of the film on the tube wall, which in turn causes a variation in temperature of the tube wall. The shields were beveled so that the condensate may flow down over them without an abrupt change in direction. This precaution helps to give correct tube-wall temperatures. The resistances of the liquid and vapor films are known to be of the same order of magnitude, and the over-all temperature drop will be large. Therefore, it is expected that any variation in the temperature of the tube wall near these shields will not have a critical effect on the calculation of the film coefficients.

Calibrations

The rate of flow of the oil was determined with a sharp-edged orifice. This orifice was calibrated at 212° and 133° F. with a petroleum oil having a viscosity of 79 seconds at 100° F.

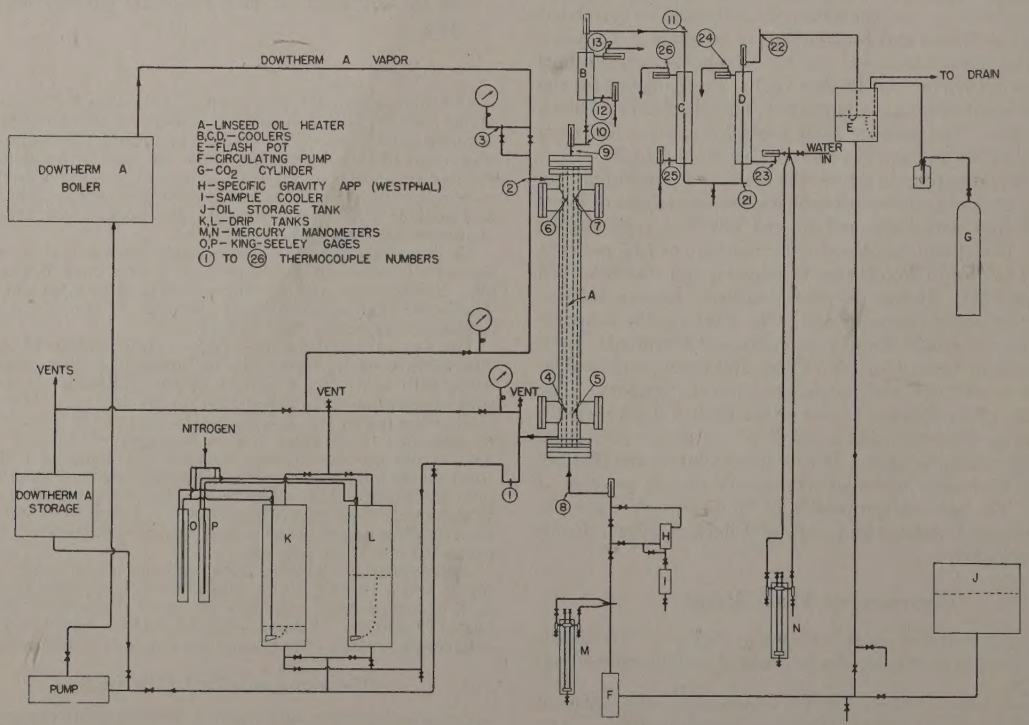

FIGURE 2. DETAILS OF THERMOCOUPLE SHIELD ATTACHMENT

This oil had approximately the same specific gravity and viscosity that the linseed oils used in this heat transfer study would have under experimental conditions at the higher temperatures.

The thermocouples used for the measurement of temperatures were calibrated in an oil bath against a calibrated resistance thermometer between the temperatures of 11° and 360° C. The temperature as measured is estimated to be accurate to ±0.5° C. The copel-X-chromel couples were checked with the boiling points of naphthalene and sulfur.

The thermocouples mounted on the linseed oil heater tubes were checked by putting Dowtherm vapor into the heater at known pressures. The inlet vapor for runs 39 to 43, inclusive, was slightly superheated; in runs 44 to 48, inclusive, it came direct from the boiler and had to be slightly wet to ensure saturation conditions of temperature for each pressure. When the pressure became constant, the temperatures of the vapor in, the condensate out, and the tube thermocouples were measured and found to be in excellent agreement.

Heat lost from the linseed oil heater by radiation was determined by putting the heater under pressure with slightly superheated Dowtherm vapor. When the pressure in the heater remained constant, the condensate and time were measured. This procedure was followed at six different pressures.

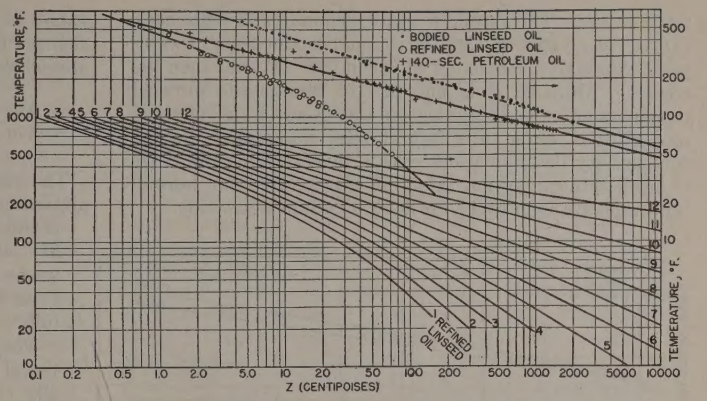
TEMPERATURE, °F.
• BODIED LINSEED OIL
○ REFINED LINSEED OIL
+ 140-SEC. PETROLEUM OIL
1 2 3 4 5 6 7 8 9 10 11 12
REFINED LINSEED OIL
Z (CENTIPOISES)
TEMPERATURE, °F.

FIGURE 3. VISCOSITY *vs.* TEMPERATURE FOR BODYING LINSEED OIL

Physical Properties of Oils

The specific heats of the petroleum oil and the bodied linseed oil were estimated with the use of the Fortsch and Whitman equation (*5*):

$$C = \frac{(t + 670)(2.10 - S)}{2030}$$

This equation was obtained for petroleum oils and should be suitable for the petroleum oil used in these experiments. It is believed that this equation is also suitable for the linseed oil. With this equation the specific heat of the pure linseed oil was calculated to be 0.703 at 550° F. This value agrees to within 6 per cent of the data published by Long, Reynolds, and Napravnik (*11*).

The viscosities of the petroleum oil, refined linseed oil, and a bodied linseed oil were determined with a MacMichael viscometer over a temperature range of 50° to 600° F. The viscometer was calibrated with water, a U. S. Bureau of Standards standard oil sample No. 4, and castor oil. The error introduced on account of the expansion of the disk plunger at 700° F. is less than the experimental error. Several fresh samples of oil were used in covering the 50–600° F. temperature range.

The data are plotted on logarithmic coördinates in Figure 3. Guide lines 1 to 12 represent linseed oil in progressive stages of polymerization or increasing bodying effect. These lines were obtained by dividing the distance between the curves for the refined and bodied linseed oil into eight equal parts. Lines 10 to 12 are extrapolations. If the viscosity of a bodied linseed oil is measured at room temperature, its viscosity can be estimated with Figure 3 at any temperature between 50° and 600° F.

The specific gravities of the petroleum oil, refined linseed oil, and a bodied linseed oil were determined with the Westphal balance equipped with an invar plummet (Figure 4).

The molecular weight was determined in order to estimate thermal conductivity. The freezing point depression method of Beckman was employed for the petroleum oil. Benzene was the solvent, and the molecular weight was found to be 744.

The molecular weight for bodied linseed oil was determined through a correlation of the data of Caldwell and Mattiello (*3*) with viscosity. Figure 5 is a plot of their values of viscosity in poises *vs.* molecular weight on a log-log chart. The curve represents viscosity determinations at 25° C. (77° F.) converted to 15.5° C. (59.9° F.) by Figure 3. The error introduced through the use of these data can have but a minor effect on the calculated value for thermal conductivity for linseed oil. For example, if the molecular weight determined is in error by about 5 per cent, the calculated thermal conductivity will be in error approximately 1 per cent.

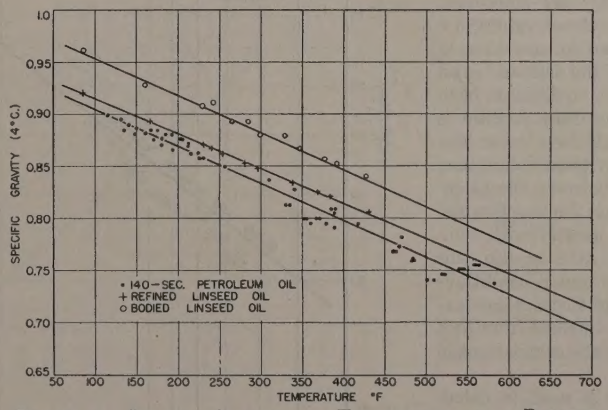

FIGURE 4. SPECIFIC GRAVITY *vs.* TEMPERATURE FOR BODYING LINSEED AND PETROLEUM OILS

Bodied linseed oil: $ds/dt = -0.000352$
Refined linseed oil: $ds/dt = -0.000334$
$(ds/dt)_{av.} = -0.000343$

Thermal Conductivity

All of the equations that have been commonly used for the correlation of heat transfer coefficients to or from fluids involve the thermal conductivity of the fluid. This offered a particular difficulty in the present case because the temperatures at which the thermal conductivities were to be evaluated were far beyond any range at which thermal conductivities of liquids have ever been measured. Within very narrow ranges the thermal conductivity of most organic liquids is known to decrease with temperature. For a long time it was believed that the thermal conductivity of water increased linearly with

temperature. The recent work of Schmidt and Sellschopp (*21*) showed, however, that the thermal conductivity of water passes through a maximum at about 260° F. and then decreases. Although it is not safe to discuss the physical properties of organic liquids on the basis of the physical properties of water, this fact at least shows that thermal conductivity determinations over a relatively narrow range of temperature may not safely be extrapolated far beyond the temperatures at which the determinations were made.

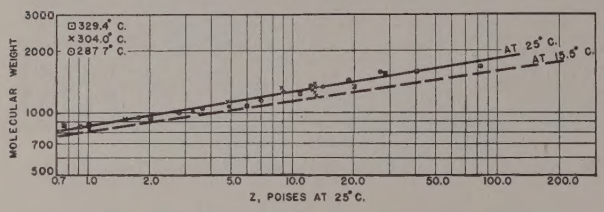

FIGURE 5. MOLECULAR WEIGHT *vs.* VISCOSITY FOR BODYING LINSEED OIL

Weber (*26*), Bridgman (*2*), and Smith (*23, 24*) proposed equations based on various physical constants for the prediction of thermal conductivities. When these equations are applied to linseed oil at 600° F., they give results differing by several hundred per cent. It is doubtful whether the authors of these equations ever expected them to be valid under such conditions. The direct determination of thermal conductivity is a difficult matter in any case and would have been so complicated at the temperatures involved in this work that it was out of the question to consider it.

The only recourse, therefore, was to try one of the above equations to see whether or not the results could be correlated on the basis of the data furnished by it, even though it was unknown whether the values of thermal conductivity so obtained were reliable or not. This does not introduce any fundamental error into the correlation as is shown by the following line of reasoning.

Experimental data, directly determined, are correlated into a single equation by the use of an arbitrary value for the thermal conductivity of the liquid, which may or may not be correct. In using the equation so determined for calculating a desired heat transfer coefficient (i. e., the reverse process from that considered in this paper), the same values of thermal conductivity, determined in the same way, are employed. Consequently, even though these values are purely empirical, they are used in one sense in developing the equation from the data and in an opposite sense in determining coefficients from the equation; thus any error they contain is neutralized. It follows that, if there is a serious error in the thermal conductivity so determined (and especially if the ratio between the calculated thermal conductivity and the true thermal conductivity should change greatly with temperature), it would not be safe to use this method at temperatures or under conditions where the numerical value of thermal conductivity to be used was greatly different from that employed in the range where the calculations in this paper were made.

When the Weber equation is used to calculate the thermal conductivity of the petroleum oil, the correlation of the data gives values which, when checked against the Dittus and Boelter equation, indicate the conductivity values to be high. If the constant in the Weber equation is changed from 0.864 to 0.342, the petroleum oil data will fall around the Dittus and Boelter line. The equations used are:

$$\text{For bodying linseed oil:} \quad k = 0.864\,(S)(C_p)^3\sqrt{S/M}$$
$$\text{For petroleum oil:} \quad k = 0.342\,(S)(C)^3\sqrt{S/M}$$

The runs on petroleum oil were made first, and then the apparatus was cleaned before introducing the linseed oil. The petroleum oil was washed out with kerosene, followed by hot solutions of trisodium phosphate, and was finally washed several times with hot distilled water. The heater tubes had some carbon residue on them. They were carefully wiped clean with cloths.

The bodying of the linseed oil took place over an interval of several hours, and the viscosity was continually changing. The time was divided into 15-minute periods. At the end of the first 15 minutes and every 30 minutes thereafter, a sample of the bodying oil was removed. Three sets of readings for pressures and temperatures, and of the mercury manometer were taken for each 15-minute run. When the viscosity had increased to such an extent that the oil velocity began to decrease, the valve for the Dowtherm vapor was closed and the flow of water to the coolers increased. When the linseed oil had cooled down to about 250° F., it was discharged from the apparatus. Fresh linseed oil was immediately introduced into the apparatus and circulated to wash out the bodied oil. The equipment was then drained, and a new charge of linseed oil pumped in to keep the heater tubes and piping filled with oil.

Smoothing and Correlation of Data

The experimental data obtained from heating the petroleum oil were used direct in the usual heat-transfer calculations. Erratic results were recognized only by comparisons among the data or later when they were correlated. The ex-

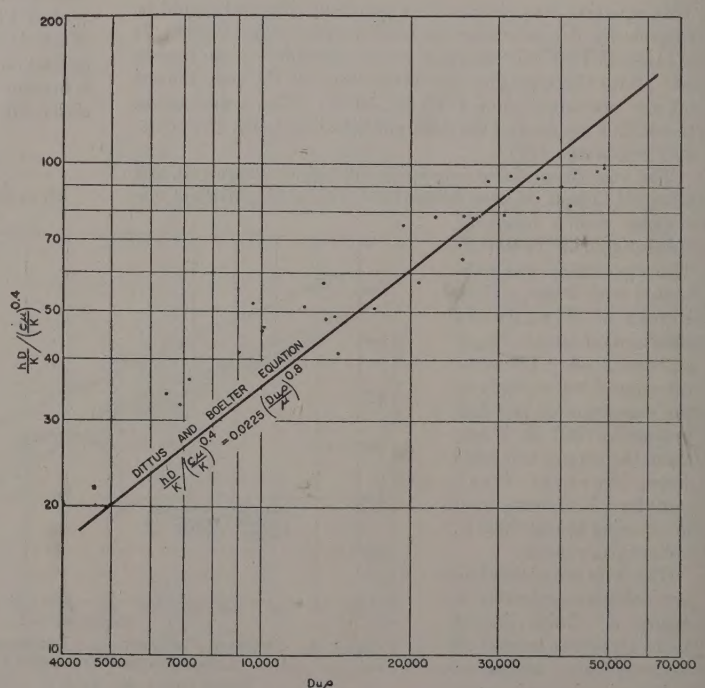

FIGURE 6. HEAT TRANSFER DATA FOR 140-SECOND PETROLEUM OIL AT 210° F.

perimental data obtained from heating the linseed oil, however, can be given a preliminary treatment to improve them. This oil was heated for periods of 2.5 to 4 hours. The readings for each 15-minute run were averaged and plotted against the average time for each interval, and smoothed values obtained from this curve. The B. t. u. transferred, vapor, tube, and oil temperatures, liquid film coefficients, and viscosity were smoothed graphically in this manner. The gross weights of Dowtherm condensate were given a more special treatment by the Running method of graphical smoothing (*20*). This was done because the rate of condensation decreased rapidly with the time of heating.

It was impossible to check the heat input by calculating that absorbed by the oil, since the temperature increase is generally only about 12° F. The accuracy of the temperature measurements was estimated to be about ±1° F., which leaves room for considerable error when the heat absorbed by the oil is calculated.

The data on linseed oil were correlated with the following Dittus and Boelter equation (*13*):

$$\frac{hD}{k} = a\left(\frac{Du\rho}{\mu}\right)^n\left(\frac{C\mu}{k}\right)^m \qquad (1)$$

The physical properties used in the correlation of the data were based on the arithmetic average of the inlet and outlet temperatures of the oil stream after mixing. This was done in preference to using the film temperature because the correlated data can be compared directly with those presented by McAdams (*12*). In addition, the data are in a more useful form for design purposes. The film temperature of the linseed oil was generally about 30° F. higher than the main body of the oil (which was about 590° F.). A correlation based on the physical properties taken at the film temperature would have given only slightly different results.

The data for the petroleum oil were plotted as $Nu/(Pr)^{0.4}$ against Re and are illustrated in Figure 6. The line representing the Dittus and Boelter equation was drawn through these data. A fairly good correlation resulted.

The values of the exponents for the Prandtl and Reynolds groups for the linseed oil runs were next obtained by trial and error as outlined by Logan, Fragen, and Badger (*10*). This correlation fixed m equal to 0.3 and n equal to 0.64 with the data defining the usual band of points, except that each set of data approximated a straight line (Figure 7).

The values for m and n seem quite low, especially n. The value of m was increased to 0.4 and $Nu/(Pr)^{0.4}$ was plotted against Re. The result was to find the data drawn into a narrow band and is illustrated in Figure 7. If lines were drawn through these data, they would have a slope equal to 0.78. This correlation of the data checks the line of the Dittus and Boelter equation (with a slope of 0.8) very well.

The value of the exponent of the Prandtl group was increased to 0.5 and the $Nu/(Pr)^{0.4}$ plotted against Re. Figure 7 illustrates the result of this operation. All the data are drawn so close together as to approximate a straight line, rather than a band. The slope of this line is 0.87. If the exponent for the Prandtl group is increased further, the correlation becomes poor again with the data falling into a band. Table I illustrates the effect of increasing the value of the exponent of the Prandtl group at a constant Reynolds number of 20,000. The oil velocity is expressed in feet per second, but the Nusselt, Reynolds, and Prandtl groups are in consistent units.

TABLE I. BODYING OF LINSEED OIL

Run No.	Oil Velocity	Nu	Pr	$Nu/(Pr)^{0.3}$	$Nu/(Pr)^{0.4}$	$Nu/(Pr)^{0.5}$	$Nu/(Pr)^{0.8}$
90	7.7	345	70.3	99.6	62.9	41.1	11.5
57	7.1	325	65.3	93.3	60.8	40.2	11.45
66a	3.75	233	31.6	94.3	58.0	41.1	14.72

The data for the bodying of linseed oil, therefore, correlate best with the following empirical equation:

$$\frac{hD}{k}\Big/\left(\frac{C\mu}{k}\right)^{0.5} = 0.00725\left(\frac{Du\rho}{\mu}\right)^{0.87} \qquad (2)$$

Sherwood and Petrie (*22*) have stated that the data on individual fluids can be correlated better with special equations than with the Dittus and Boelter equation. This equation was tried on the data on petroleum oil and some of the data of Morris and Whitman (*19*). A slight improvement was noted, but not enough to warrant its use.

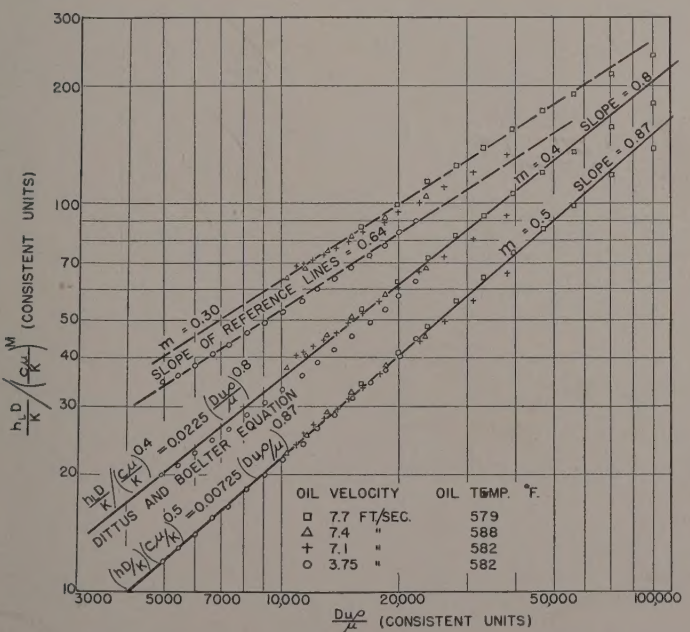

FIGURE 7. HEAT TRANSFER DATA FOR BODYING LINSEED OIL

Conclusion

The liquid-film heat transfer coefficients have been determined for a 140-second petroleum oil at 210° F. and for a bodying linseed oil in forced convection at high temperatures. The data on petroleum oil cover a temperature range of 350° to 600° F.; that of the linseed oil, 570° to 600° F. These data were obtained from a vertical heater, equipped with two 10-foot, 0.75-inch i. d. nickel tubes. All the results correlated well with the Dittus and Boelter equation:

$$\frac{hD}{K} = 0.0225\left(\frac{Du\rho}{\mu}\right)^{0.8}\left(\frac{C\mu}{k}\right)^{0.4} \qquad (1A)$$

The degree of correlation is indicated by the following table, which gives the agreement between the calculated and the experimental values of the liquid film coefficients:

Per Cent of Data Correlated to:	Bodying Linseed Oil	Petroleum Oil
	%	%
± 5%	67	15
±10%	98	38
±20%	100	59

Equation 1A was checked between the following limits:

	Bodying Linseed Oil	Petroleum Oil
Nusselt No.	153–520	250–600
Reynolds No.	5,000–90,000	4,500–50,000
Prandtl No.	14–160	50–370
Oil velocity, ft./sec.	3.75–7.7	4–7.8
Viscosity, lb./hr./ft.	1.0–8.5	1.5–10.7

The linseed oil data were even better expressed by the empirical equation. It can be recommended only for bodying linseed oil with the thermal conductivity estimated by Weber's equation. All the physical properties of the oils were taken at the average temperature of the main body of the heated oil:

$$\frac{hD}{k} = 0.00725 \left(\frac{Du\rho}{\mu}\right)^{0.87} \left(\frac{C\mu}{k}\right)^{0.5} \qquad (2)$$

Nomenclature

Hr. = hours of heating
a = constant in Dittus and Boelter equation
D = diam., ft.
C = sp. heat, B. t. u./(lb.)(° F.)
C_p = sp. heat at constant pressure
k = thermal conductivity, B. t. u./(hr.)(sq. ft.)(° F./ft.)
ρ = density, lb./cu. ft.
M = molecular weight
S = sp. gr. referred to water at 4° C.
Z = viscosity (by MacMichael viscometer), centipoises
Z_t (av.) = av. viscosity at temp. t ° F. (values smoothed graphically), centipoises
μ = viscosity = 2.42 Z_t (av.), lb./(hr.)(ft.)
u = oil velocity, ft./hr.
Nu = Nusselt group $h_L D/k$
Re = Reynolds group $Du\rho/\mu$
Pr = Prandtl group $C\mu/k$

Acknowledgment

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